Interamerican Journal of Psychology, 1976, *10*, 65-78

THE ROLE OF MATHEMATICAL EDUCATION IN A CHANGING SOCIETY[1]

Elizabeth M. Williams
Education Sub-Committee-British National Committee on Mathematics

The subject of my talk does not refer to any particular country. In fact it is almost universal, for there is hardly a society that is not undergoing dramatic and far-reaching change, technological, social or economic. Rapid change has produced among many people all over the world a sense of restless uncertainty, of fear that human control is slipping, of personal helplessness. This is remarkable at a time when man has achieved an astonishing control of much of his physical environment through his use of the ideas and techniques of mathematics, the natural sciences and technology. What has come to be realized as crucial to our obvious failure to use similar insights and skills in the complex problems of a world interrelated in its economic, political and social systems. We can look at pictures of the surface of Venus but we cannot adequately feed the peoples of India. We create atomic power but we cannot manage the earth's money. We have made great strides in genetics but we destroy and maim thousands (or even millions) of human beings in a year through disease, lack of care, violence or preventable accidents.

This dichotomy in man's development of mastery has caused anxiety among many scientists as was evident in two fairly recent conferences. British Commonwealth Scientists with sociologists met in Jamaica to discuss the social significance of science and mathematics. Some progress was made in recommending experimental courses in school which would show the use of mathematics in making surveys of social or economic activities but little impact has been made at national levels. There followed an International Congress at Maryland, U.S.A., to explore possibilities in organising integrated studies in mathematics and science on themes of social importance. A Committee was appointed in order to maintain contact with experiments resulting form the discussions at the conference. However, the part played by mathematics remains a subsidiary one, and little consideration has been given to the role that mathematics might have in finding solutions to our organisational problems. But even survival is now seen to be at stake and this risk

---

[1] This article is based on a conference given by professor Elizabeth Williams at the Sociedad Científica Argentina in November 1975. The Editor of the Interamerican Journal of Psychology requested Prof. Williams permission to publish it. As our readers probably know, Prof. Williams is a very distinguished educator who has contributed in many ways and in many countries to the development of new educational approaches to the study and teaching of Mathematics. In this article Prof. Williams presents a challenging view of how the teaching of Mathematics can help solve some of the riddles emerging from a rapidly changing society.

The Editor

will surely drive us to look afresh at what education *could* do. To cure our world-wide ills we need an educated population which will understand, even in simple terms, the complex structures required to supply the necessities of life and to develop the human potential that science has revealed. We can note in passing that it is estimated that 12 million children receive no education at all; we have a long way to go before we can assume that nearly every human being understands sufficiently the environment on which he depends. But we have mentioned the word *structures* and this brings us to our main question, what can *mathematics* do to assist in the solution of the new social problems?

It may be suggested that school mathematics is itself in such confusion between old and new that it is not in a condition to offer *any* remedy. The speed of recent mathematical and psychological developments has brought them too suddenly to be fully absorbed by teachers. There have been two eruptions. First there has been the impact of new ideas about the essential bases of mathematical instruction from about 1960; e.g. the ideas of cardinal numbers and simple logical operations through the use of sets; and secondly there is the culmination of a movement -prompted by psychologists and teachers — to rescue the mass education of children from the superficial formalism which was restricting it. Instead experiences were to be planned for the learners which would bring a keener awareness of the world outside the classroom and of how to *live* in it— confidently, if ardously. A record of traffic conditions outside school is an example of such an experience. The time has surely come now to blend what has proved of undoubted value in the new with those traditional ideas which are an essential part of basic mathematics. In such a blend it should be possible to provide a unified programme able to give pupils a confident personal mastery of the mathematical ideas relevant to contemporary problems.

There are people who ask a more searching question. At national and international conferences on mathematics education I have heard it asked: Has mathematics *any* personal educational value? Can it not be regarded as a body of *techniques* from which a selection can be made to suit the needs of the individual learner? This is to misunderstand the nature of mathematics and what if offers to scientists and to citizens. A valuable book has just been published, it is by that well-known figure in international mathematics Hans Freudenthal (1972) of Utrecht, Holland, and is called *Mathematics as an Educational Task.* In it he shows mathematics in its social setting and places it firmly in the context of general education as an *activity* related to the real world. It is the *nature* of this activity, seen in various forms, that we have to identify.

A Report was recently published in Britain by the Mathematical Association (1972) called *Mathematics, 11-16* to indicate that it is intended to cover the whole range of ability during the compulsory years of secondary schooling. It boldly asserts that *patterns* is the essence of mathematics – *recognizing* patterns, *expressing* patterns in other forms, *creating* patterns. It considers various kinds of pattern numerical, spatial, symbolic, logical, expressed in movement, etc. *The relations embodied in the patterns are the subject matter of mathematical study.*. It is the emphasis on relatedness with its implications of order, structures and transformation which gives mathematics its importance as an element in education. It gives to the individual learner a picture of a world in which prediction, and sometimes control, are in large measure, over a wide field, open to manking. Yet man is not imprisoned within a fully known system. Discovery and invention are still possible to him either alone or with companions. As one example of the linking together of number, space and symbol patterns, we can cite the Pythagorean theorem about squares on the sides of a right-angled triangle; the numerical statement about the sums of pairs of ordered square numbers, the drawing of a set of right angled triangles on the same hypotenuse, the statement $x^2 + y^2 = a^2$ and the circle which results from this equation when its graph is drawn in the conventional position and showing the symmetries about the axes; these all demonstrate the close connection between *spatial* and *numerical* or *algebraic* relations.

Another interesting book relevant to our topic was published in Britain by Oxford University Press in 1972 under the tittle "Starting Points" (Banwell et al, 1972). It is quite different from any other book on school mathematics and I want to quote from it: "It is sometimes said that mathematics is essentially problem-solving. This can be misleading, for a problem is seldom well-defined."

After mentioning the "doodling" that helps to clarify the situation comes this sentence, "The importance of mathematics in *education* lies in *process* rather than in *product*". In other words *it is in identifying and connecting relations that the educational value lies.*

Clearly, learning mathematics implies building up a memory bank of relations and relational systems as a source for solving *future* problems. The larner must therefore encounter experiences exhibiting relations in a memorable way, possibly in investigations that are not neccessarily problems but may involve the assembling of facts that *reinforce* or *modify* a known pattern. There may be no "answer" to his inquiry but he has the satisfaction of having acquired a usable insight.

Since a relation may occur in a variety of forms (e.g. the inclusion relation) we often have a choice in the way in which we exhibit it. This is an important consideration for the teacher, because we know that there is diversity among children in regard to the preferred form of imagery. Since success in learning mathematics depends in part on the ease of recall of certain patterns, presentation in the preferred imagery may help many slow new learners to *have* the success which brings self confidence.

One of the welcome characteristics of recent innovations in school mathematics is the range of types of diagrams in which mathematical patterns, information, movements, etc., can be exhibited. This has made it possible to introduce topics that have hitherto seemed too advanced for less gifted pupils. The use of vectors, matrices, Venn diagrams, Cartesian co-ordinates, etc., has made clear to ordinary young people mathematical ideas of powerful use in science, organization or industrial processes. It is this element of *power* in mathematics which can give strength to the will to learn or to use what has been learned.

Recently the schools Council in Britain has been responsible for a project called "Mathematics for the Majority", that is for the broad band of medium ability. It has been based on the commoner uses of mathematics and on practical activities. It has resulted in a new consideration of the levels of understanding that teachers can expect to find. Plato, in The Republic, first described the characteristics of the levels which he saw as marked on a line of increasing insight. Today we can distinguish three stages in the grasping of a mathematical argument or logical process. At the simplest level the pupil follows each successive-step and is sure of its validity but he cannot reproduce the entire process. At the second level the whole process is understood and can be reproduced. At the highest level the learner can *discover* the process for himself and produce it for others to see. We realise that in considering mathematics for a whole population there will be very many in the first category. The value of an insight into ideas and processes whose foundations may have been forgotten is very high in a decision-making democracy, where well-founded opinion is vital. An understanding of the methods of the calculus can produce a respect for mathematical procedures even when its techniques have never been mastered. The emphasis is on the partial *success*, rather than the partial *failure*.

The first stage in solving a mathematical problem is the search for a clue, looking in the memory bank to see whether anything there offers a starting point. It may be that very quickly a number of pieces of information or some related facts fall into place, and the solution *appears*. But the path first selected may lead to no progress at all. It is here that the quality of flexibility is all-important. A new start *must* be made. George Polya (1948) author of the well-known book "How to solve it", urges that an *entirely new* approach should be attempted to show the problem in a new light. He gives interesting examples but I will quote an instance from an English classroom where the chance of a good solution was missed through persisting in the first line of attack. The class was presented with 5 by 3 rectangle divided into unit squares. The problem was to find a general formula for the number of squares a diagonal passes through. Groups of the children drew a great many different rectangles, made many counts, examined special cases, tried a variety of formulae, until they found the formula "one less than the sum of the two sides", (3 + 5 — 1, in this case) with the *proviso* that "if the diagonal passes through a 'corner' of a square, the corner must be counted as a square". The problem was solved but without a very deep understanding. If the tactics had been changed and the diagram considered as a network of lines, *some* of which must be *crossed* to reach the corner of the rectangle, a *direct* solution would have been *evident*. Moreover the *"corner"* is seen as the crossing *of two lines at once*. The clue to the general formula comes from 1 + (3—1) + (5—1) for the given special case. This *willigness to examine* a problem *anew*. from a different point of view, is not only a good mathematical technique; it is a most valuable quality in a member of a decision-making society.

There are certain mathematical topics that are particularly useful in a society in which finances, institutions, population and industry distributions, the extension of education are all subject to new planning. Elementary statistics can follow easily in the primary school from the classifications and comparisons which occur with sets. The idea of an *average* grows early from sets of personal measurements; *mean, mode* and *median* are all used for recording and for comparisons with measurements in other classes. Developments take place as number skills increase. The subject of statistics can be burdensome and boring in later stages unless the inquiries are kept realistic and the results are *used* in interesting ways. The social value of these studies is evident from the *misuse* of statictics, sometimes deliberately. The story is still told in England that after a survey of children's reading abilities had been published in a newspaper, a reader wrote to say how scandalous it was that nearly half the children were below average! Recently we have been troubled by disputes about wages increases: when an agreement is made with members of a factory or a trade union that the *average* increase shall be *not more than* 6 pounds it is claimed that no one must receive *less* than 6 pounds increase. When no one is content to be shown as below average it is desirable to use other ways of stating a norm and to *teach more statistics* to ordinary young people, particulary in relation to their own achievements and interests.

Rates of growth have a strong contemporary interest — in regard to industrial and agricultural products, the value of currency, the physical measurements of children, animals and plants, national populations, etc. Young people are often astonished to see the effects of a compound rate even when the rate for a unit of time, say a month or a year, si as small as 4 or 5%. Graphical illustrations and tables of values of figures obtained from current reports can lead to a better understanding, say, of the effect of interest rates on prices of goods and the profits of a tradesman. At this time the idea of a growth factor and the use of logarithms can be appreciated, particularly in *identifying the growth factor.* Scientists, mathematicians and economists can fruitfully work toghether in this field. For the mathematician himself some valuable study of other sequences can develop which will have a bearing on other phenomena.

In these threatening days when so many men have become aware of themselves as members of a vast and rapidly growing family of human beings, dependent almost entirely on the resources of the finite earth, the study of mathematics can make two great contributions: it can show the patterns and structures on the basis by which mankind can organize, increase, and distribute our resources; these principles can also lead to new *inventions* which will make man more secure. From the personal point of view it can hearten the younger generation by revealing to it the immense and unique power that man has developed through his capacity for mathematical thinking, and in which the individual himself can have a share. With this outlook there might be something of a *real* revolution in the stanging of mathematics in the educational system -mathematics might even become popular.

References

Banwell, C.S.; Saunders, K.D. & Tahta, D.G. *Starting Points.* Oxford: Oxford University Press, 1972.

Freudenthal, H. *Mathematics as an Educational Task.* Boston: D. Reidel, 1972.

Polya, G. *How to Solve It.* Princeton, New Jersey: Princeton University Press, 1948.

The Mathematical Association of Great Britain. *Mathematics 11-16.* London: Bell, 1972.