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SOME EFFECTS OF NON-NORMAL DISTRIBUTION SHAPE ON THE MAGNITUDE OF THE PEARSON PRODUCT MOMENT CORRELATION COEFFICIENT[1]

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Abstract. All correlational statistics must be interpreted with consideration given to certain constraints. The use of the Pearson product moment correlation coefficient as a descriptive measure of relationship necessitates the assumption that the correlated variables be distributed bivariately normal. This assumption is probably not met in some applications of the Pearson product moment correlation coefficient as a descriptive statistic. This study was undertaken to establish some empirical evidence as to the consequences of this action. The results indicate that lack of bivariate normality can have a sizeable effect on the magnitude of obtained values of the Pearson product moment correlation coefficient.

Resumen. Estadísticas correlacionales deben ser interpretadas bajo ciertas restricciones. El uso del coeficiente de correlación de Pearson como medida de relación requiere la suposición de que las variables correlacionadas se destribuyen bivariantemente normales. Esta suposición probablemente no es observada en muchas aplicaciones del coeficiente de correlación de Pearson como estadística descriptiva. Este estudio examinó las consecuencias de la falla de observarse la normalidad bivariante de las variables. Los resultados indican que esta falla tiene efectos considerables sobre la magnitud del coeficiente de correlación de Pearson que se obtiene.

As long as the Pearson product moment correlation coefficient has been in use it has been recognized that shape of the bivariate distribution influences the magnitude of this statistic. This influence may only be important for some uses of the Pearson product moment correlation coefficient, however.

Carroll (1961, p. 349) stated that the two commonly recognized uses of correlational methods are to serve "(1) as a basis for prediction from one variable to another, or from a set of variables to one or more dependent variables, and (2) as a way of measuring something called 'relationship' between variables." In the behavioral sciences the use of correlation as a measure of "relationship" frequently occurs in psychometrics (especially in test construction) and in applications of factor analysis. Binder (1959) indicated that for such uses of correlation the appropriate model is the bivariate normal model. This study is an attempt to investigate the Pearson product moment correlation technique as a measure of the relationship between variables when certain assumptions of the bivariate normal model are not met.

In the paper in which he published his mathematical development of a correlational technique, Karl Pearson (1895) called attention to the inaccuracy of the product moment correlation coefficient when the variables are not bivariate normally distributed. As a solution to the problem he proposed that a correlational technique should be developed which could be used with distributions with any degree of skewness.

Yule (1897) derived a correlational method based only on the assumptions inherent in the least square procedure of curve fitting. In this manner he was able to arrive at several descriptive interpretations of such a product moment method when no distributional assumptions are made. The interpretation which is relevant for this paper involves the effect produced on the magnitude of the correlation coefficient when the condition of bivariate normality does not occur. Yule was able to show that although product moment correlation coefficients take on values between plus and minus one regardless of distributional assumptions the maximum absolute value of the correlation coefficient must be less than one if the true regression is not linear.

In a presentation of factors which affect correlation coefficients McNemar (1969, pp. 186-187) discussed some effects evidenced by skewed marginal distributions which could result from the joint distribution of the population being other than bivariate normal. He stated that lack of normality, particularly due to skewness in the marginal distributions, was probably indicative of non-linear regression lines, heterogeneity of array variances, and skewed array distributions. Further, when the marginal distributions are skewed in the same direction and the true (population) correlation is negative, the range of the attainable values of r will generally be somewhat reduced at the lower extreme (minus one). In addition, when the marginal distributions are skewed in opposite directions and the true (population) correlation is positive, the range of the attainable values of r will generally be somewhat reduced at the upper extreme (plus one). For example, when the marginal distribution on the abcissa is negatively skewed to the extreme and the marginal distribution on the ordinate is positively skewed to the extreme, the majority of cases in such a scatter-diagram will have to appear in the lower right quadrant. Since for perfect positive correlation, all the cases would have to fall on a diagonal from the lower left to the upper right, it would be impossible to discover a correlation approaching plus one from such a bivariate population. McNemar also states that the magnitude of the distortion of the correlation will be greater when the "degree of relationship" is high than when it is low. Thus it appears that the bias in the magnitude of the value of correlation can be seen as a function of the skewness of the marginal distributions.

This study was conducted in order to investigate some of the effects produced on the values of the Pearson product moment correlation coefficient when populations are not distributed in a bivariate normal manner. Skewness of the marginal distributions is used as the indicator of departure from the bivariate normal.

The effects of marginal skewness on sampling distributions of the Pearson product moment correlation coefficient have been investigated by E. S. Pearson (1929, 1931, 1932); Chesire, Oldis and E. S. Pearson (1932); Dunlap (1931); Baker (1930); Rider (1932); Norris and Hjelm (1960); and Veldman (1969). In general these studies reveal that the presence of marginal skewness is an indication that the sampling distribution of the Pearson product moment correlation coefficient is disrupted. However, since these studies utilizing the sampling distribution to attempt to determine the effects of lack of bivariate normality on obtained correlation coefficients assume that the relationship present in a non-normal bivariate distribution was adequately described by the Pearson product moment correlation coefficient, this investigation took a different approach.

This investigation was performed by generating bivariate normal populations for particular degrees of relationship using the mathematical definition of the ideal bivariate surface utilizing a large digital computer. Each of the resulting bivariate distributions was then repeatedly systematically skewed using power transformations. After transformation, correlation coefficients were computed. Tabulations of these correlation coefficients for particular marginal distribution shapes and original correlations are indicative of the effect of lack of bivariate normality on the Pearson product moment correlation technique. Skewness and kurtosis of the marginal distributions as defined by Fisher (1958) were used as measures of distribution shape.

PROCEDURE

The steps followed in the conduct of this study were 1) generation of bivariate normal populations with a specific degree of correlation, 2) transformation of these data to induce skewness, and 3) calculation of a correlation coefficient on the skewed data.

The distributions were generated by using the mathematical expression of the bivariate normal surface. This expression in terms of the variables X and Y is

$$H = Ae^{-P}$$

Where H is the height of the surface

$$A = \frac{N}{2\pi \sigma_X \sigma_Y \sqrt{1 - r^2}}$$

$$P = \frac{1}{2(1 - r^2)} \left(\frac{x^2}{\sigma_X^2} + \frac{y^2}{\sigma_Y^2} - \frac{2rxy}{\sigma_X \sigma_Y} \right)$$

N is total number of X,Y pairs

x and y are deviations of particular values of X and Y from their respective means

X and Y are standard deviations of the X and Y values

and r is the correlation between the X and Y variables.

The procedure followed in the generating process consisted of obtaining values of H by causing the X and Y values to vary between definite limits. The bivariate surface was thus defined by the values of height (*H*) for appropriate values of X and Y. Populations of 100,000 X, Y pairs were generated by causing the X and Y variables to assume all integer values between one and 99. The marginal distributions for X and Y had means of 50.0 and standard deviations of ten. Ten populations representing correlations of ±.90, ±.70, ±.50, ±.30 and ±.10 were generated.

Since in this procedure the values of X and Y were only allowed to assume integer values, a discrete variable was used to approximate values of a continuous variable. For this reason and due to the non-infinite size of the population, values of skewness and kurtosis for the X and Y distributions depart slightly from the values expected for an ideal univariate normal distribution.

In order to produce skewness, non-linear power transformations were performed on the values of the X and Y distributions. This transformation consisted of raising the X or Y values of the marginal distributions to some decimal power. Thirteen values of power ranging from 1.0 to 10.6 with intervals of .8 between successive values were used.

It can be seen that this transformation accomplishes a rescaling of the X and Y values such that they no longer form an equal interval scale. Since frequency of occurrence of the particular X or Y value in the marginal distribution is unchanged but the interval between the values is changed, the transformation changes the shape of the distribution. When the exponential term for the transformation is greater than one, positive skewness results; and when the exponential term of the transformation is less than one, but greater than zero, negative skewness results. Theoretically, in this manner marginal distributions with any degree of skewness may be obtained.

However, practical problems arise when exponents between one and zero are used to produce negative skewness. These problems arise because as the exponent approaches zero, the standard deviation approaches the mid-range interval size. As the size of the standard deviation equals or is less than the interval size, a correction for grouping should be applied to the standard deviation and all statistics involving higher moments of the distribution. However, procedures to correct for grouping error assume equal interval scales and normal distributions. To circumvent these problems, negatively skewed distributions were produced through the use of linear transformation on the positively skewed distributions. This linear transformation produced negatively skewed distributions by taking the mirror image of the positively skewed distributions. The problems of grouping were thus avoided when generating positively skewed distributions because the standard deviation remained much larger than the mid-range interval size as the exponent increased.

After the shape of the marginal distributions had been altered, the remainder of the procedure consisted of quantifying the shape of the transformed distributions and computing the correlation for the transformed data. Any change in the correlation coefficient from its value previous to the transformation process can be attributed to the effect of transformation on the correlation coefficient.

The study was conducted by obtaining observed correlation coefficients for each of the 10 bivariate populations for each of the pairings of the 13 values of positive and the 13 values of negative skewness. Each of the observed correlation coefficients representing some particular value of marginal skewness of the X variable and some particular value of marginal skewness for the Y variable for some bivariate population with a particular correlation will be referred to as belonging to one of two cases. *Case One* involves the effects of distribution shape on calculated r when both marginal distributions are skewed in the same direction. *Case Two* involves the effects of distribution shape on calculated r when the marginal distributions are skewed in opposite directions.

RESULTS

The results are presented according to the two configurations of direction of skewness of the marginal distributions in tabular form and in graphical form. Information on the parameters of a transformed distribution is given in Table 1. This table shows the mean, standard deviation, skewness, and kurtosis of a univariate distribution which results from the transformation of a normal distribution by the values of power shown. The normal distribution consisted of 100,000 cases and had a mean of 50.0 and a standard deviation of 10.0 before transformation.

Case One

Tables 2 through 11 present values of observed r for differing amounts of skewness for marginal distributions of X and Y when both are skewed in the same direction, i.e., when the skewness for X and Y have the same sign. Each table represents a distribution with a different degree of relationship. The table for negative skewness of X and Y would be identical to

TABLE 1

Changes in Distribution Parameters Caused by a Power Transformation on the Ordinate Values of a Normal Distribution

Power	Mean	Standard Deviation	Skewness	Kurtosis
1.00	5.0000×10	1.0000×10	.0000	-.0004
1.80	1.1736×10^{3}	4.1288×10^{2}	.4756	.2769
2.60	2.8311×10^{4}	1.4191×10^{4}	.9339	1.3105
3.40	6.9582×10^{5}	4.5611×10^{5}	1.4123	3.2091
4.20	1.7438×10^{7}	1.4258×10^{7}	1.9408	6.3440
5.00	4.4500×10^{8}	4.4123×10^{8}	2.5497	11.4154
5.80	1.1551×10^{10}	1.3641×10^{10}	3.2738	19.6437
6.60	3.0470×10^{11}	4.2348×10^{11}	4.1554	33.1135
7.40	8.1606×10^{12}	1.3243×10^{13}	5.2476	55.3373
8.20	2.2174×10^{14}	4.1797×10^{14}	6.6162	92.1373
9.00	6.1087×10^{15}	1.3329×10^{16}	8.3422	152.9443
9.80	1.7051×10^{17}	4.2978×10^{17}	10.5223	252.5963
10.60	4.8198×10^{18}	1.4017×10^{19}	13.2689	413.6408

the table for positive skewness of X and Y. Therefore only one table is necessary to present the results of both negative and positive skewness for a particular value of original *r*. (See notes at the bottom of each table for information needed to interpret the table.)

Examination of Tables 2 through 11 shows that higher absolute values of skewness produced the greatest reduction in observed *r*'s from their original value. Also the reduction in observed *r*'s is much greater for the negative values of original *r*'s than for the positive values. Although a table is not provided for an original *r* of zero, results not shown demonstrate that all observed *r*'s for an original *r* of zero are also zero.

TABLE 2

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Greatest Curtailment of Calculated *r* for an Actual *r* of +.90

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4121	1.9404	2.5488	3.2718	4.1514	5.2398	6.6018	8.3163	10.4774	13.1941
.0000	.9000	.8942	.8787	.8558	.8269	.7934	.7564	.7167	.6752	.6325	.5894	.5465	.5044
.4756		.8780	.8532	.8221	.7862	.7468	.7051	.6618	.6177	.5734	.5296	.4868	.4454
.9338			.8205	.7827	.7414	.6979	.6532	.6079	.5628	.5184	.4751	.4335	.3937
1.4121				.7397	.6945	.6484	.6020	.5560	.5111	.4675	.4256	.3858	.3483
1.9404					.6468	.5992	.5523	.5067	.4627	.4206	.3807	.3432	.3082
2.5488						.5511	.5046	.4600	.4176	.3776	.3400	.3051	.2727
3.2718							.4592	.4163	.3759	.3382	.3031	.2708	.2411
4.1514								.3754	.3374	.3022	.2698	.2401	.2131
5.2398									.3019	.2693	.2396	.2125	.1880
6.6018										.2395	.2123	.1878	.1657
8.3163											.1878	.1656	.1458
10.4774												.1458	.1281
13.1941													.1123

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also.

TABLE 3

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Least Curtailment of Calculated *r* for an Actual *r* of ±.90

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4121	1.9404	2.5488	3.2718	4.1514	5.2398	6.6018	8.3163	10.4774	13.1941
.0000	.9000	.8942	.8787	.8558	.8269	.7934	.7564	.7167	.6752	.6325	.5894	.5465	.5044
.4756		.8988	.8928	.8782	.8566	.8292	.7971	.7614	.7227	.6820	.6400	.5974	.5549
.9338			.8958	.8895	.8754	.8546	.8282	.7972	.7624	.7247	.6848	.6435	.6016
1.4121				.8912	.8846	.8707	.8505	.8249	.7947	.7607	.7238	.6847	.6442
1.9404					.8852	.8783	.8645	.8447	.8196	.7901	.7569	.7207	.6825
2.5488						.8781	.8708	.8571	.8375	.8129	.7839	.7514	.7161
3.2718							.8699	.8622	.8484	.8290	.8049	.7765	.7447
4.1514								.8606	.8526	.8387	.8196	.7957	.7680
5.2398									.8503	.8420	.8281	.8091	.7857
6.6018										.8392	.8305	.8166	.7978
8.3163											.8271	.8182	.8043
10.4774												.8143	.8052
13.1941													.8008

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also.

TABLE 4

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Greatest Curtailment of Calculated *r* for an Actual *r* of +.70

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4122	1.9407	2.5494	3.2731	4.1541	5.2453	6.6123	8.3355	10.5113	13.2514
.0000	.7000	.6955	.6834	.6656	.6431	.6171	.5883	.5574	.5251	.4919	.4584	.4250	.3921
.4756		.6847	.6670	.6443	.6176	.5881	.5565	.5234	.4896	.4556	.4217	.3884	.3561
.9338			.6446	.6178	.5879	.5559	.5225	.4883	.4539	.4197	.3862	.3537	.3224
1.4122				.5878	.5556	.5219	.4875	.4530	.4187	.3850	.3524	.3211	.2913
1.9407					.5218	.4872	.4525	.4181	.3845	.3519	.3206	.2908	.2626
2.5494						.4524	.4179	.3842	.3517	.3204	.2906	.2626	.2362
3.2731							.3842	.3516	.3204	.2907	.2627	.2364	.2120
4.1541								.3204	.2908	.2628	.2367	.2123	.1898
5.2453									.2629	.2368	.2126	.1901	.1695
6.6123										.2127	.1903	.1697	.1509
8.3355											.1698	.1511	.1341
10.5113												.1342	.1188
13.2514													.1049

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also.

TABLE 5

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Least Curtailment of Calculated *r* for an Actual *r* of +.70

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4122	1.9407	2.5494	3.2731	4.1541	5.2453	6.6123	8.3355	10.5113	13.2514
.0000	.7000	.6955	.6834	.6656	.6431	.6171	.5883	.5574	.5251	.4919	.4584	.4250	.3921
.4756		.6973	.6910	.6782	.6602	.6379	.6121	.5837	.5532	.5212	.4884	.4553	.4223
.9338			.6901	.6824	.6689	.6506	.6284	.6028	.5747	.5445	.5131	.4808	.4482
1.4122				.6795	.6705	.6564	.6378	.6155	.5901	.5623	.5326	.5017	.4701
1.9407					.6659	.6559	.6412	.6223	.6000	.5748	.5473	.5182	.4880
2.5494						.6499	.6390	.6237	.6046	.5823	.5573	.5303	.5019
3.2731							.6317	.6200	.6042	.5850	.5628	.5382	.5118
4.1541								.6117	.5993	.5832	.5638	.5419	.5177
5.2453									.5901	.5771	.5608	.5415	.5198
6.6123										.5672	.5538	.5373	.5182
8.3355											.5433	.5296	.5131
10.5113												.5186	.5048
13.2514													.4935

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also.

TABLE 6

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Greatest Curtailment of Calculated *r* for an Actual *r* of ±.50

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4123	1.9408	2.5497	3.2736	4.1552	5.2472	6.6156	8.3412	10.5207	13.2665
.0000	.5000	.4968	.4882	.4754	.4594	.4408	.4202	.3981	.3750	.3513	.3274	.3035	.2801
.4756		.4904	.4789	.4637	.4455	.4252	.4032	.3801	.3563	.3322	.3081	.2847	.2612
.9338			.4650	.4478	.4280	.4065	.3836	.3600	.3359	.3119	.2880	.2648	.2422
1.4123				.4290	.4080	.3857	.3624	.3386	.3147	.2910	.2678	.2452	.2236
1.9408					.3864	.3636	.3403	.3167	.2933	.2702	.2477	.2261	.2055
2.5497						.3408	.3177	.2946	.2718	.2496	.2282	.2076	.1881
3.2736							.2951	.2727	.2508	.2296	.2092	.1898	.1715
4.1552								.2512	.2303	.2102	.1911	.1729	.1558
5.2472									.2106	.1917	.1737	.1569	.1411
6.6156										.1740	.1574	.1418	.1272
8.3412											.1420	.1276	.1143
10.5207												.1145	.1023
13.2665													.0913

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also.

TABLE 7

Effect of the Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Least Curtailment of Calculated *r* for an Actual *r* of ±.50

Skewness on X	Skewness on Y												
	.0000	.4756	.9338	1.4123	1.9408	2.5497	3.2736	4.1552	5.2472	6.6156	8.3412	10.5207	13.2665
.0000	.5000	.4968	.4882	.4754	.4594	.4408	.4202	.3981	.3750	.3513	.3274	.3035	.2801
.4756		.4968	.4911	.4810	.4673	.4506	.4316	.4108	.3887	.3657	.3422	.3185	.2950
.9338			.4883	.4807	.4694	.4548	.4377	.4184	.3976	.3756	.3528	.3296	.3064
1.4123				.4757	.4667	.4543	.4391	.4216	.4022	.3814	.3597	.3373	.3147
1.9408					.4599	.4497	.4365	.4208	.4030	.3837	.3632	.3418	.3201
2.5497						.4415	.4303	.4165	.4005	.3827	.3636	.3434	.3227
3.2736							.4211	.4091	.3949	.3787	.3611	.3423	.3227
4.1552								.3990	.3865	.3721	.3560	.3386	.3203
5.2472									.3758	.3630	.3485	.3326	.3156
6.6156										.3518	.3389	.3245	.3090
8.3412											.3275	.3146	.3005
10.5207												.3032	.2905
13.2665													.2792

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also.

TABLE 8

Effect of Degree of Skewness on Calculated r when the Configuration of the Marginal Distribution Produces the Greatest Curtailment of Calculated r for an Actual r of +.30

Skewness on X	Skewness on Y												
	.0000	.4756	.9339	1.4123	1.9408	2.5497	3.2738	4.1554	5.2476	6.6162	8.3422	10.5223	13.2689
.0000	.3000	.2981	.2929	.2852	.2756	.2645	.2521	.2389	.2250	.2108	.1964	.1821	.1680
.4756		.2950	.2888	.2803	.2699	.2582	.2454	.2318	.2177	.2034	.1890	.1748	.1608
.9339			.2818	.2726	.2617	.2496	.2365	.2228	.2087	.1945	.1803	.1663	.1527
1.4123				.2629	.2516	.2393	.2262	.2125	.1986	.1846	.1708	.1572	.1440
1.9408					.2402	.2279	.2148	.2014	.1878	.1742	.1607	.1476	.1350
2.5497						.2156	.2028	.1897	.1765	.1633	.1504	.1379	.1259
3.2738							.1903	.1776	.1649	.1524	.1401	.1282	.1168
4.1554								.1654	.1533	.1414	.1297	.1185	.1078
5.2476									.1418	.1305	.1196	.1091	.0990
6.6162										.1199	.1097	.0999	.0906
8.3422											.1002	.0911	.0825
10.5223												.0827	.0748
13.2689													.0676

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being negative and the sign of all calculated r's in the body of the table should be negative also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being positive and the sign of all calculated r's in the body of the table should be positive also.

TABLE 9

Effect of Degree of Skewness on Calculated r when the Configuration of the Marginal Distribution Produces the Least Curtailment of Calculated r for an Actual r of +.30

Skewness on X	Skewness on Y												
	.0000	.4756	.9339	1.4123	1.9408	2.5497	3.2738	4.1554	5.2476	6.6162	8.3422	10.5223	13.2689
.0000	.3000	.2981	.2929	.2852	.2756	.2645	.2521	.2389	.2250	.2108	.1964	.1821	.1680
.4756		.2973	.2932	.2865	.2778	.2673	.2556	.2429	.2294	.2154	.2013	.1871	.1729
.9339			.2902	.2845	.2766	.2670	.2560	.2439	.2309	.2174	.2036	.1897	.1758
1.4123				.2797	.2728	.2640	.2538	.2424	.2301	.2172	.2038	.1903	.1768
1.9408					.2667	.2588	.2494	.2388	.2273	.2150	.2022	.1892	.1762
2.5497						.2518	.2433	.2335	.2227	.2111	.1990	.1866	.1741
3.2738							.2356	.2266	.2166	.2058	.1944	.1827	.1707
4.1554								.2185	.2093	.1993	.1886	.1776	.1663
5.2476									.2009	.1917	.1818	.1715	.1609
6.6162										.1832	.1741	.1646	.1547
8.3422											.1658	.1570	.1478
10.5223												.1489	.1405
13.2689													.1328

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being positive and the sign of all calculated r's in the body of the table should be positive also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being negative and the sign of all calculated r's in the body of the table should be negative also.

TABLE 10

Effect of Degree of Skewness on Calculated *r* when the Configuration of the Marginal Distribution Produces the Greatest Curtailment of Calculated *r* for an Actual *r* of ±.10

Skewness on X	Skewness on Y												
	.0000	.4756	.9339	1.4123	1.9408	2.5497	3.2738	4.1554	5.2476	6.6163	8.3423	10.5224	13.2691
.0000	.1000	.0994	.0976	.0951	.0919	.0882	.0840	.0796	.0750	.0703	.0655	.0607	.0560
.4756		.0986	.0968	.0941	.0908	.0871	.0829	.0785	.0739	.0691	.0644	.0596	.0550
.9339			.0949	.0922	.0889	.0851	.0810	.0766	.0720	.0673	.0626	.0580	.0534
1.4123				.0895	.0862	.0825	.0784	.0741	.0696	.0650	.0604	.0559	.0515
1.9408					.0830	.0793	.0753	.0711	.0668	.0623	.0579	.0535	.0492
2.5497						.0757	.0719	.0678	.0636	.0593	.0551	.0509	.0468
3.2738							.0682	.0643	.0602	.0562	.0521	.0481	.0442
4.1554								.0605	.0567	.0528	.0490	.0452	.0415
5.2476									.0531	.0494	.0458	.0422	.0388
6.6163										.0460	.0426	.0393	.0360
8.3423											.0394	.0363	.0333
10.5224												.0334	.0306
13.2691													.0280

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being negative and the sign of all calculated *r*'s in the body of the table should be negative also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being positive and the sign of all calculated *r*'s in the body of the table should be positive also.

TABLE 11

Effect of Degree of Skewness on Calculated r when the Configuration of the Marginal Distribution Produces the Least Curtailment of Calculated r for an Actual r of +.10

Skewness on X	Skewness on Y												
	.0000	.4756	.9339	1.4123	1.9408	2.5497	3.2738	4.1554	5.2476	6.6163	8.3423	10.5224	13.2691
.0000	.1000	.0994	.0976	.0951	.0919	.0882	.0840	.0796	.0750	.0703	.0655	.0607	.0560
.4756		.0988	.0972	.0948	.0917	.0881	.0841	.0797	.0752	.0705	.0657	.0610	.0563
.9339			.0958	.0935	.0905	.0870	.0831	.0789	.0745	.0699	.0652	.0606	.0560
1.4123				.0913	.0885	.0852	.0815	.0774	.0731	.0686	.0641	.0596	.0551
1.9408					.0859	.0827	.0792	.0753	.0711	.0669	.0625	.0581	.0538
2.5497						.0797	.0764	.0727	.0687	.0647	.0605	.0563	.0521
3.2738							.0732	.0697	.0660	.0621	.0581	.0541	.0502
4.1554								.0664	.0629	.0593	.0555	.0517	.0480
5.2476									.0596	.0562	.0527	.0491	.0456
6.6163										.0530	.0497	.0464	.0431
8.3423											.0466	.0435	.0405
10.5224												.0407	.0378
13.2691													.0352

Note. This table can be interpreted in two ways, depending on the signs of the skewness for the X and Y marginal distributions. If both the X and Y marginal skewnesses have either positive or negative signs (both are skewed in the same direction), the true relationship should be interpreted as being positive and the sign of all calculated r's in the body of the table should be positive also. If the X and Y marginal skewnesses have opposite signs (X and Y are skewed in opposite directions), the true relationship should be interpreted as being negative and the sign of all calculated r's in the body of the table should be negative also.

Case Two

Tables 2 through 11 also present values of observed *r* for different skewnesses of X and Y when the signs of the skewness for X and Y are different. The difference in the interpretation of Tables 2 through 11 for Case One and Case Two is only a matter of sign. A note at the bottom of each table explains the proper interpretation.

Examination of Tables 2 through 11 shows that observed *r* is most reduced from its original value by the higher absolute values of skewness. Also, the reduction in observed *r* is greater for the positive values of original *r* than for the negative values.

Graphic Presentation

Figures 1 through 5 present a summary of the information in Tables 2 through 11. Each figure represents a combination of two tables. The figures show families of curves of observed *r*'s for a particular original *r* plotted against skewness of Y. Various levels of the skewness of X define each member of the family. Similar to the tables, the figures have two interpretations which differ only by sign. If the figures are interpreted as is, they represent positive true relationships. If all signs are reversed, the figures represent negative true relationships.

It is important to note that each figure is a combination of both Case One and Case Two for a particular population value of *r*. Case One consists of positive values of skewness for both X and Y. Case Two consists of negative values of skewness for Y, with positive values of skewness for X. Thus Case One and Case Two form families of continuous curves for constant values of skewness of X, since curves in both cases for a particular value of skewness of X have a common point at zero skewness for Y.

Though the curves are not shown in the figures, Case One would also consist of negative values of skewness for both X and Y, and Case Two would consist of positive values of skewness for Y, with negative skewness for X. These configurations would also form families of continuous curves for constant values of skewness of X, joining at zero value of skewness for Y. These families of curves were omitted because they are mirror images of the families of curves shown. In order to interpret the figures according to the configurations not shown, the reader need only establish signs for positive or negative relations as necessary, and then change the signs for all values of skewness.

The figures indicate the effect that distribution shape, as measured by skewness, has on the Pearson product moment correlation coefficient. A significant feature of the figures is their general similarity. Each figure shows that when both marginal distributions are normal, observed *r* is equal to the original *r*. Each curve for constant skewness of X has a maxi-

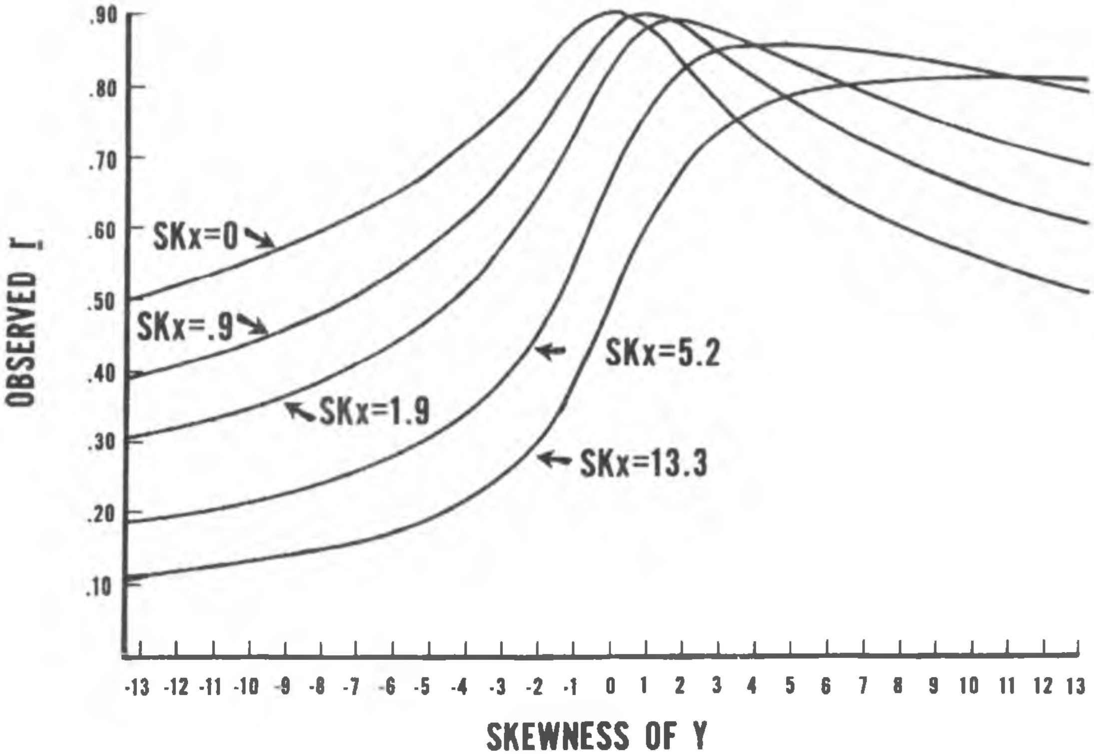

Figure 1.--Family of curves showing observed r vs. skewness of Y for various values of skewness on X and an original r of +.90.

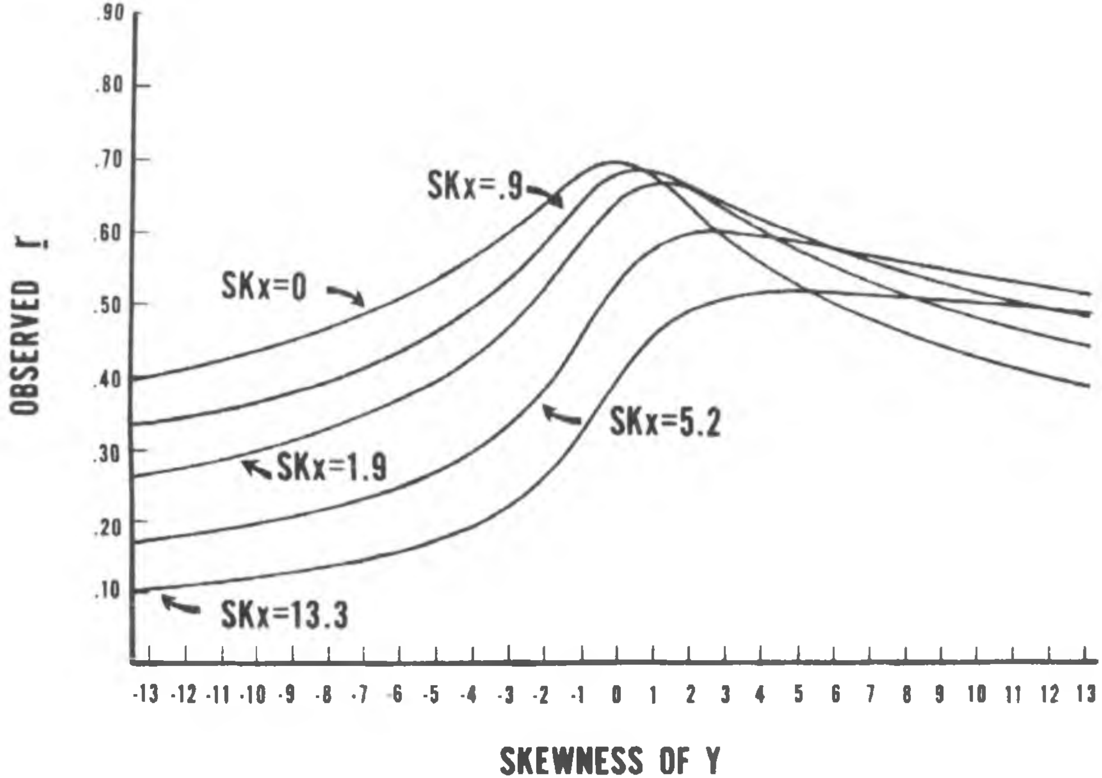

Figure 2.--Family of curves showing observed r vs. skewness of Y for various values of skewness on X and an original r of +.70.

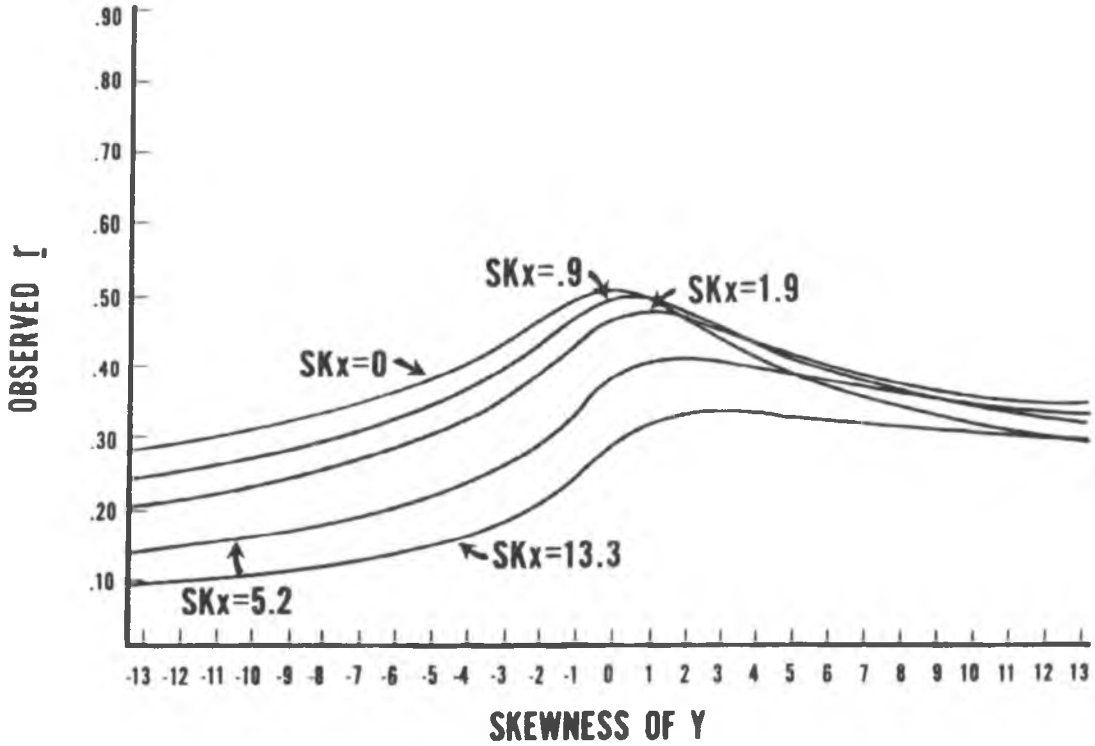

Figure 3.--Family of curves showing observed r vs. skewness of Y for various values of skewness on X and an original r of +.50.

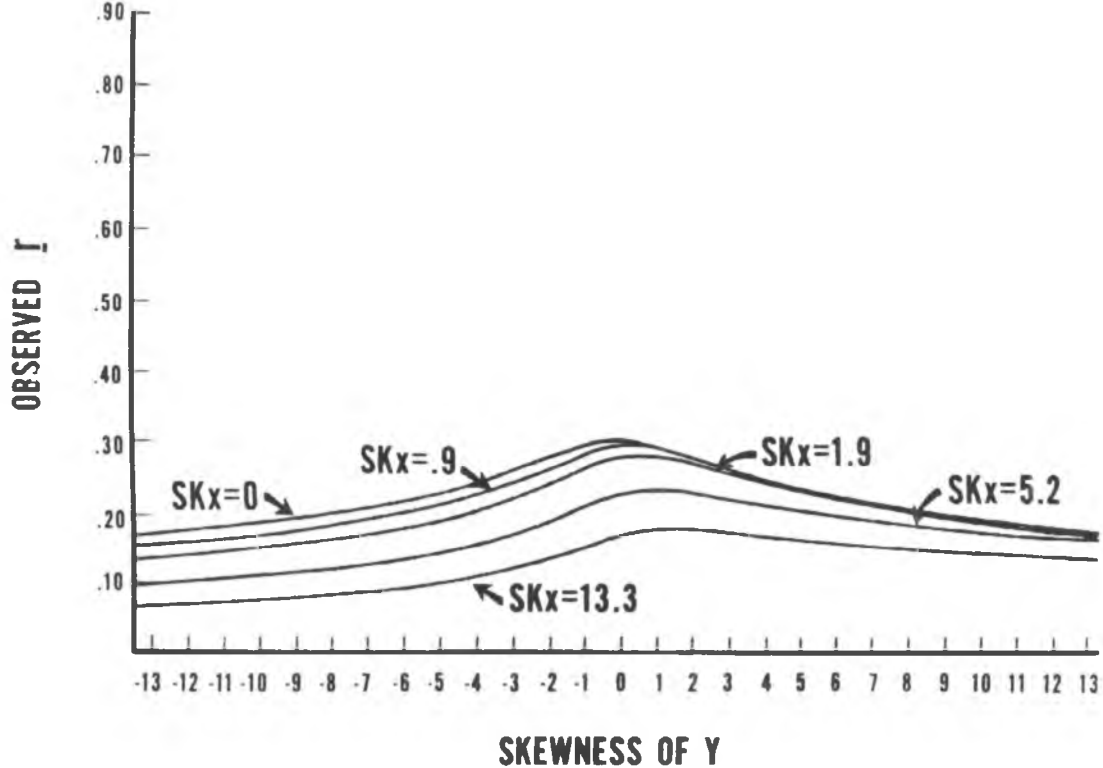

Figure 4.--Family of curves showing observed r vs. skewness of Y for various values of skewness on X and an original r of +.30.

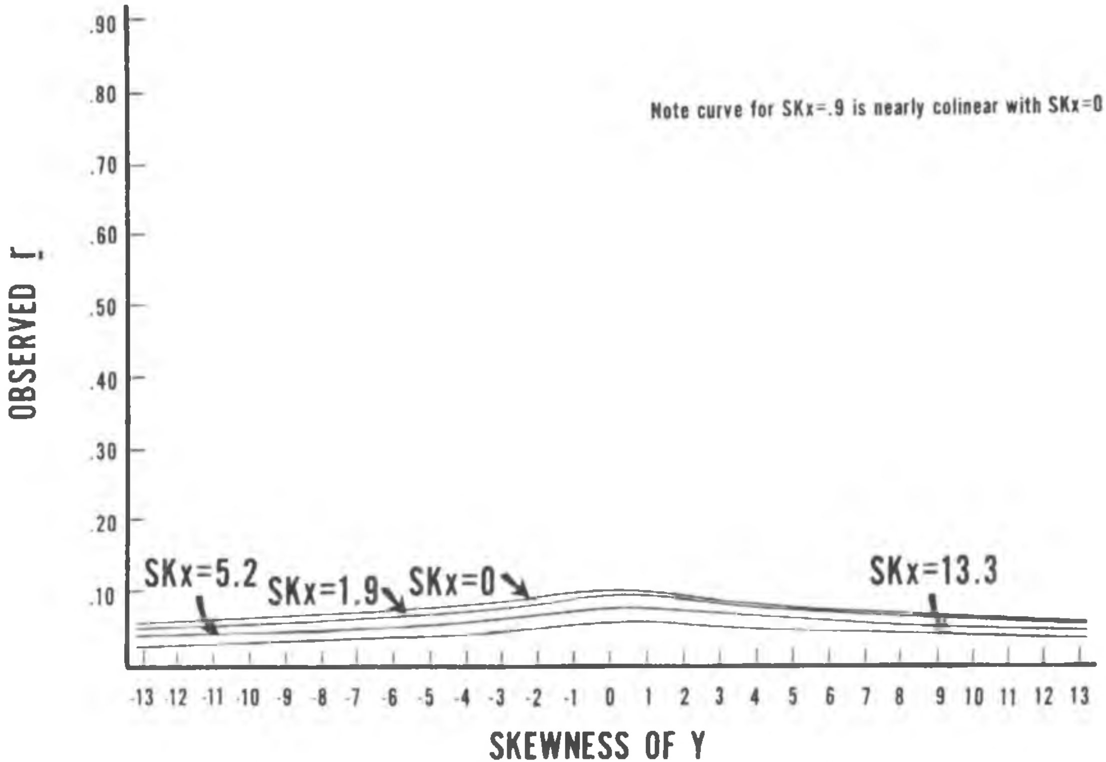

Figure 5.--Family of curves showing observed r vs. skewness of Y for various values of skewness on X and an original r of +.10.

mum point for a value of skewness on Y nearly equal to the value of skewness of X for the curve.

The results of this study are summarized in terms of the two configurations of marginal distribution shape and sign of the true relationship.

I. Case One — both marginal distributions skewed either positively or negatively but in the same direction.

A. Positive original relationship. The departure of the obtained correlation from the original value is small for small magnitudes of skewness for both X and Y but becomes fairly large for larger values of marginal skewness for either or both X and Y. However, when one marginal distribution is highly skewed, the departure of the obtained correlation from the original value is large and nearly uniform regardless of the shape of the other marginal. When both marginals are skewed, the least bias occurs when both marginal distributions are skewed equally.

B. Negative original relationships. The departure of the obtained correlation from the original value is small for small magnitudes of marginal skewness for both X and Y but becomes increasingly extreme for larger values of marginal skewness for either or both X and Y.

II. Case Two — both marginal distributions are skewed either positively or negatively but in opposite directions.

A. Positive original relationship. The departure of the obtained correlation from the original value is small for small magnitudes of marginal skewness for both X and Y but becomes increasingly extreme for larger values of marginal skewness for either or both X and Y.

B. Negative original relationship. The departure of the obtained correlation from the true value is small for small magnitudes of marginal skewness for both X and Y but becomes fairly large for larger values of marginal skewness for either or both X and Y. However, when one marginal distribution is highly skewed, the departure of the obtained correlation from the original value is large and nearly uniform regardless of the shape of the other marginal distribution. When both marginals are skewed, the least bias occurs when both marginal distributions are skewed equally.

DISCUSSION

The concept of correlation as a descriptive index represents an attempt to quantify relationship between variables. Mathematically the implementation of this concept involves fitting some function to a joint distribution of values from the variables. The "goodness" of the fit is taken as indication of the relationship existing between the variables, and is quantified as a correlation coefficient. The magnitude of such a correlation coefficient is inversely related to the magnitude of the error which results from fitting the mathematical function to the joint distribution. A particular application of the concept of correlation representing a specified function and a specific number of variables is a correlational model.

In such a procedure for quantification of relationships between variables, mathematical and logical constraints are implicit in the derivation and interpretation of a correlation coefficient. These constraints have major implications for application of correlational technique. When the nature of the data does not correspond to the constraints of a particular correlational model, the value of the correlation coefficient will probaly be distorted and incorrect conclusions about the relationship between the variables can result. A lack of correspondence between the data and the model can occur in two general ways: (1) the mathematical function fit to the data may be inapprpriate, and (2) although the mathematical function fit to the data is appropriate, the data may not be distributed as assumed in the model. For some applications the appropriateness of a model can be relatively evident. One such situation occurs for the descriptive use of the Pearson product moment correlational model.

When the correlational model is of the Pearson product moment type, the joint distribution represents values from only two variables, and the function fit to the bivariate distribution is linear. If the distribution of the data is bivariate normal, the Pearson product moment correlation coefficient represents a "good" description of the relationship between the variables. When the data are not distributed bivariately normal, the Pearson product moment correlation technique probably represents a biased or "less good" description of the association between the variables. This study represents an attempt to quantify how much "less good" the Pearson product moment correlation coefficient is as a measure of relationship when the two variables are not bivariate normally distributed.

Non-normality in bivariate distributions which previous experience with similar variables would lead an investigator to expect to be distributed normally is probably indicative that some quantification error has disrupted the process by which values of one or both the variables were obtained. Carroll (1961) suggested that errors of quantification can be classified as (1) errors of scaling (i.e.. censoring and/or use of a non-equal interval metric), (2) errors of scale-dependent selection (i.e., restriction in range) and (3) errors of measurement. The particular error of scaling, involving use of a non-equal interval metric, has important implications for interpreting this study.

If the amount of inequality of the intervals of a metric is uniform across the possible range of values (i.e., the amount of the inequality increases or decreases by a constant amount for succeeding values), it is possible to conceive of the consequences of the use of such a scale. However, if the amount of inequality of the intervals is not uniform such a general conception becomes impossible. The use of such a scale which has uniform unequal intervals can cause the scaling of the obtained values to be a nonlinear transformation of the true scale, and bivariate use of such disrupted scales can cause the apparent regression lines for two variables to be nonlinear when the true regression lines for the variables are linear. Since the type of transformation used in this study caused a uniform rescaling of the variables which destroyed the equality of the intervals, a strict interpretation should only permit this study to be viewed as an investigation of correlational bias which will be found in bivariate distributions which are non-normal because the scales of measurement are characterized by uniformly unequal intervals.

However, since it can be shown that correlations in normal bivariate distributions can "best" be represented by linear functions, it is reasonable to assume that correlations in most non-normal bivariate distributions can "best" be described by non-linear functions. And since the type of transformation used in the study can be seen to cause the true (actual) regression lines to be non-linear, an alternative interpretation of the study is possible. Thus a less strict interpretation could allow this study to be viewed as an investigation of the amount of bias which will be found for correlations in bivariate distributions which are non-normal because the true relationship between the variables is non-linear for a correlational model which assumes a linear relationship. However, only the effects of lack of bivariate normality due to the true relationship being non-linear as a result of a power transformation have been investigated.

The results of this study show that the amount of bias, the magnitude of the true relationship, and the degree and configuration of marginal skewness are related in a complex manner. However, the bias appears to always represent an underestimation of the value of the true relationship. In general the bias is least when both marginal distributions are nearly normal and greatest when both marginal distributions are highly skewed. The magnitude of the bias depends upon the configuration of the marginal distribution shapes and sign of the true relationship. When the true relationship is positive and the marginal distributions are skewed, the magnitude of the bias is much greater when the marginal distributions are skewed in opposite directions than when they are skewed in the same direction. When the true relationship is negative and the marginal distributions are skewed, the magnitude of the bias is much greater when the marginal distributions are skewed in the same direction than when they are skewed in opposite directions.

The results of this study should be of interest to investigators who use factor analysis or test construction techniques based on correlations between scaled scores or item responses when the item responses are considered to be estimates of a value on a continuum. In these cases it should be assumed (Binder, 1959) that the distributions of scores or item responses are normally distributed for the populations being investigated. This assumption would appear to be untenable, however, when factors are operating which cause the distributions of scores or item responses to be skewed. In such cases the results of the factor or item analysis which are obtained would be distorted due to the dependence of these techniques on undistorted correlation coefficients.

The results of this study could be useful for an investigator who wishes to estimate the amount of bias induced by lack of bivariate normality for the population he should expect, when he uses a Pearson product moment correlation coefficient as a measure of relationship. To estimate this bias an investigator need only enter the appropriate tables or figures with some knowledge of the suspected population correlation and the values of skewness for both his variables and obtain the curtailed correlation by interpolation. This curtailed correlation is the value he would expect to obtain if he computed the correlation for the entire population or the average of the values he would obtain if he secured correlations from multiple samples of fixed size from this population. In addition, if the investigator were to conclude that skewness were a problem and he desired to transform his data, Table 1 could provide an estimate of an exponential value for a power transformation.

To summarize, it has long been recognized that the Pearson product moment correlation technique leads to biased correlation coefficients when the data and the assumptions of the model used do not correspond. The magnitude of this bias for the descriptive correlational model has been the concern of this study when the variables do not meet the assumptions of the model because of a non-equal interval scaling metric or alternatively because the true relationship between the variables was non-linear. Results of the study indicate that in all cases this bias represents an underestimation of the true correlation. The magnitude of the bias is relatively small for minor deviations from bivariate normality but becomes quite large for extreme deviations from bivariate normality. The results of this study should be useful for investigators who utilize descriptive correlations with variables which are univariately skewed (because of difficulty, preference or other factors) to estimate the magnitude of the bias they should expect when they investigate the relationship between such variables. However, this study was conducted for a simulated population, so investigators who use correlations as descriptive statistics in research with skewed variables utilizing samples must consider the difficulties of estimating correlations for populations as well as bias due to the non-normality of the bivariate distribution.

The author urges that the following delimitations be entertained by readers of this paper:

1. The procedure for the power transformation did not allow for skewness and kurtosis to be controlled independently. Thus some of the discovered bias could be due to the effects of marginal kurtosis or to the joint effect of the marginal skewness and kurtosis rather than to marginal skewness alone.
2. Perhaps the use of marginal distribution shape as an indicator of lack of bivariate normality obscures other important effects which might be evident if a bivariate measurement of lack of bivariate normality had been used.

REFERENCES

Baker, George A. The significance of the product-moment coefficient of correlation with special reference to the character of the marginal distribution. *Journal of the American Statistical Association*, 1930, 25, 387-396.

Binder, Arnold. Considerations of the place of assumptions in correlational analysis. *American Psychologist*, 1959, 14, 504-510.

Calkins, Dick S. Some effects of systematic bias on the Pearson product moment correlation coefficient. Unpublished Master's Thesis, The University of Texas at Austin, 1969.

Carroll, John B. The nature of the data, or how to choose a correlation coefficient. *Psychometrika*, December 1961, 26, 4, 347-372.

Chesire, Leone, Oldis, Elena, and Pearson, Egon S. Further experiments on the sampling distribution of the correlation coefficient. *Journal of the American Statistical Association*, 1932, 27, 212-128.

Dunlap, Hilda Frost. An empirical determination of means, standard deviations and correlation coefficients drawn from rectangular populations. *The Annals of Mathematical Statistics*, 1931, 2, 66-81.

Fisher, Ronald A. *Statistical Methods for Research Workers*. New York: Hafner Publishing Company, Inc., 1958.

McNemar, Quinn. *Psychological Statistics*. New York: John Wiley and Sons, Inc., 1969.

Norris, Raymond C. and Hjelm, Howard F. *An Empirical Investigation of the Effects of Nonnormality upon the Sampling Distribution of the Product Moment Correlation Coefficient*. Nashville, Tennessee: George Peabody College for Teachers, 1960.

Pearson, Egon S. Some notes on sampling tests with two variables. *Biometrika*, 1929, 21, 336-360.

Pearson, Egon S. The test of significance for the correlation coefficient. *Journal of the American Statistical Association*, 1931, 26, 128-134.

Pearson, Egon S. The test of significance for the correlation coefficient: some further results. *Journal of the American Statistical Association*, 1932, 27, 424-426.

Pearson, Karl. Mathematical contributions to the theory of evolution — III. regression, heredity, and panmixia. *Philosophical Transactions of the Royal Society of London*, 1896, 187A, 253-318.

Rider, Paul R. On the distribution of the correlation coefficient in small samples. *Biometrika*, 1932, 24, 382-403.

Veldman, Donald J. Empirical tests of statistical assumptions. The Research and Development Center for Teacher Education, The University of Texas at Austin, 1969.

Yule, G. U. On the theory of correlation. *Journal of the Royal Statistical Society*, 1897, 60, 821-854.

NOTES

1. The contributions of the late Harvey Dingman, who supervised this study in connection with the author's M.A. thesis, are gratefully acknowledged. The author is now an Assistant Professor in the Department of Educational Psychology and Guidance at the University of Texas at El Paso.

2. The computer utilized was a CDC 6600 located at The University of Texas at Austin. The computer programs for this study can be found in Calkins (1969).